St. Nicholas

Maths Calculation Strategies

Progression through written methods

The purpose of this booklet is to provide guidance and information about the types of calculation methods that children are being taught, and are using up to the end of Key Stage 2.

Much has changed in the teaching and learning of maths over the past few years. The calculation methods used by children today are, in many cases, different from those used by adults when they were at school. This can cause anxiety, with parents and carers unsure whether or not they should teach children particular methods.

As a general rule, if your child brings home some maths work which involves calculations:

- Ask them to explain how they would solve this at school, and to explain to you the methods they have been taught. Use this booklet to help.
- If your child is unable to explain their method, or unsure about what to do, the best advice is to contact your child's teacher.

The calculation methods taught today gradually build on the children's understanding over a period of time. The aim is to teach children calculation methods which they understand, can use correctly, and can use confidently to solve problems.

The calculation policy is organised according to the age stage expectations as set out in the National Curriculum 2014. However, it is vital that children are taught according to the stage that they are currently working at, being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure to move on. At St Nicholas we will teach the children the appropriate method for their ability, understanding and confidence.

This may mean a child in Year 5 may be still using methods from Year 3. Similarly, a child in Year 2, may begin to work on Year 4 methods.

As teachers we are constantly assessing children informally every lesson, your child will be working on the method best for them within their group.

All Strategies

All of the methods detailed in this booklet can be used to solve calculations involving:

- money
- measures (g, kg, cm, m, km, ml, l)

Just remember children will not move onto the next stage if:

- they are not ready
- they are not confident

<u>When solving calculations:</u>
Children are encouraged to estimate their answers before calculating.
Children are encouraged to check their answers after calculation and see if the answer makes sense.
Check with an inverse operation (addition/subtraction) (multiplication/division)

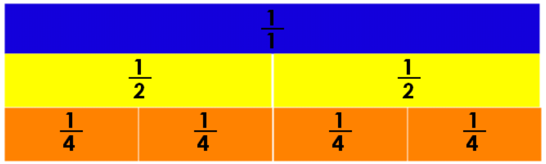
Children must learn to use the appropriate strategy when solving different problems. Sometimes a written method is the most appropriate; sometimes a mental 'counting on' method is more appropriate. Encouraging children to make these choices is a key step in a child's learning journey.

Fractions and Division

Making the link between division and fractions is very important. This allows children to apply division to a wider range of situations. These links will be made whenever children are taught fractions.
Children are taught to understand that 'fractions are division in disguise!'

e.g. $\frac{1}{2}$ of 50 = 50 ÷ 2

$\frac{1}{10}$ of 80 = 80 ÷ 10

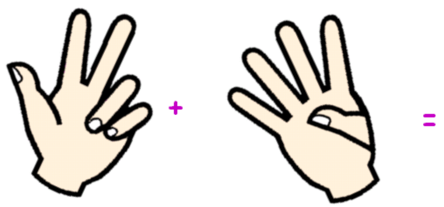


Amazing Addition

Starting Out

Year R

Children begin by recognising numbers and matching numbers to different amounts of objects. They count objects one at a time to find a total which ensures they don't count an object more than one. It is called one to one correspondence.

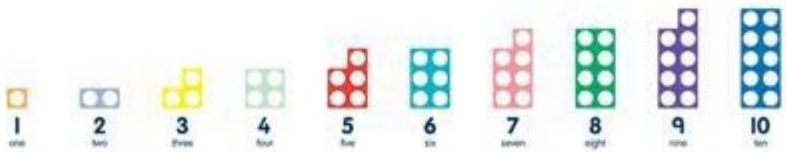


Children use fingers as first counting apparatus and continue to use them for supporting the solving of further calculations.

Great for counting in 1s up to ten or 10s up to 100 - any more and there aren't enough fingers and it all becomes muddled!!

Children use other objects for counting and adding too!
They can combine the groups and then count how many there are in total.

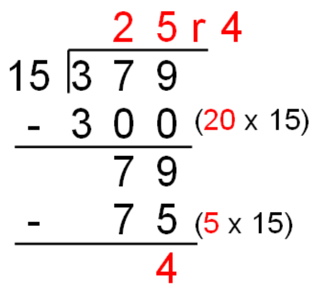
Children can also use Numicon to add numbers together, using the holes to count. They can make number sentences and recognise the colours and shapes of different numbers.



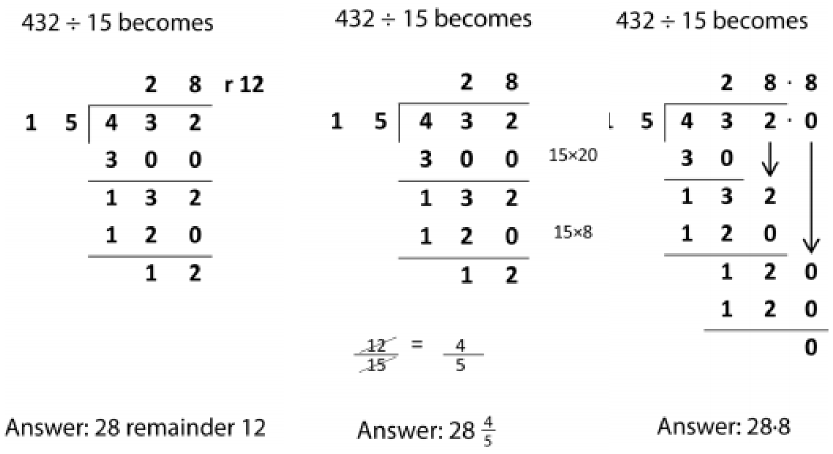
Long Division

Year 5/6

Children can use long division to solve HTU divided by TU calculations. It is based on the same method as vertical chunking but children must take away the biggest chunk possible each time.



Children move on to calculating remainders as fractions and then continuing the calculation to find decimals.



Division vertically by chunking

Year 4/5

Once children are secure at using multiples of ten to solve division, they move onto the vertical method of recording. This is the same process but just recorded vertically. Children take away multiples of ten at once.

```
      3 9 r 7
8  | 3 1 9
 -   2 4 0  (30 x8)
       7 9
 -     7 2  (9 x8)
         7
```

Short Division

The efficient method for division comes next but only when children are secure with the chunking method. (number line or vertical) Start by dividing the number into the most significant digits first. Place value is lost using this efficient method. It is important to estimate the size of an answer before starting.

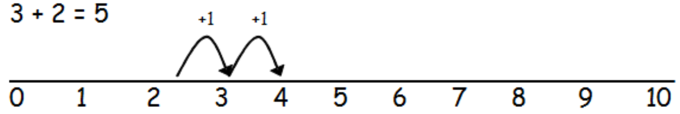
```
     1 3                    3 9 r 7
6 | 7 ¹8              8 | 3 ³1 ⁷9
```

Children need to be secure in this method and when they are, they will move onto solving problems and writing the remainder .

Using a number line

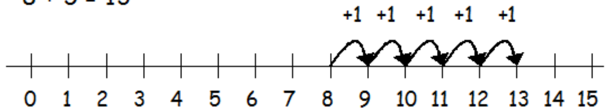
Year 1/2

3 + 2 = 5



Children move onto using pre-drawn number lines to count on in ones initially under ten, then bridging ten, then higher numbers.

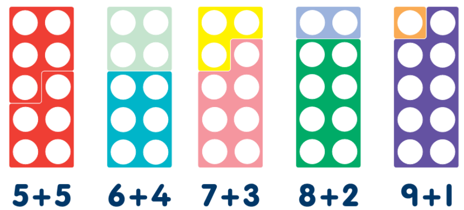
8 + 5 = 13

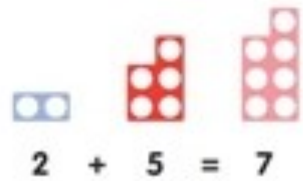


Children will still continue to use cubes, counters, bead strings and fingers to solve calculations.

Using Numicon

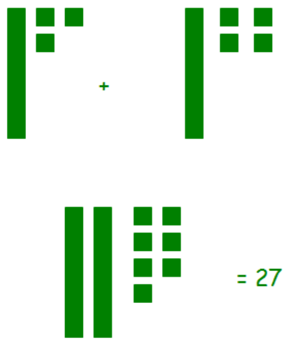
Children use Numicon to solve additions and identify their number bonds to 10.

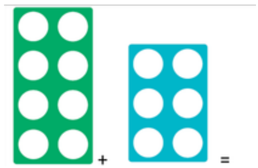




Using Numicon, Diennes, Place Value Cards and Hundred Squares

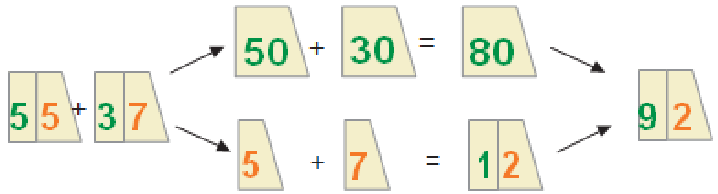
Any year to support learning

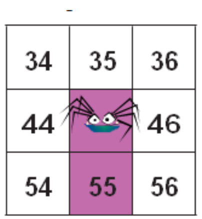




Children use equipment to make a TU number then combine the tens and units and add it up.

Children also use place value cards to add numbers together by splitting (partitioning) into tens and units.
They continue to use place value cards throughout school to support calculations.





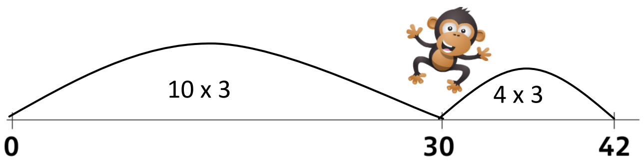
45+10 =55

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Division on a number line by chunking (chunky chimp)

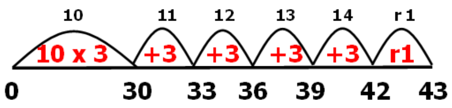
Year 3/4

When dividing larger numbers, children are able to calculate bigger jumps. They begin with a multiple of ten, using chunky chimp.
42 ÷ 3 = 13



Children will begin to do the second jump as a 'chunk'.
As children become more confident they can take away more than one 'chunk' of 10 x
Children begin to solve division with remainders.

43 ÷ 3 = 14 r 1



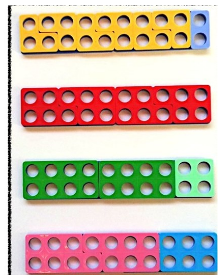
Children can check results using Numicon or use Numicon to solve these problems practically.

20 ÷ 3 = 6r2

20 ÷ 5 = 4

20 ÷ 8 = 2r4

20 ÷ 7 = 2r6

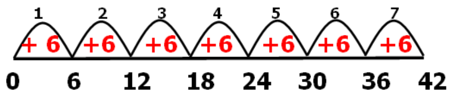


Division by counting up on a number line

Year 2/3

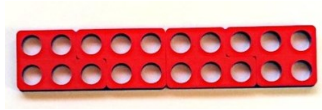
Once children have secured their understanding of grouping objects they move onto using a number line to solve division calculations.

$42 \div 6 = 7$



Children count up from 0 in steps of the number they are dividing by. They can use Numicon and overlay the steps being counted in over the answer.

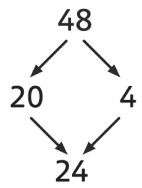
$20 \div 5 = 4$

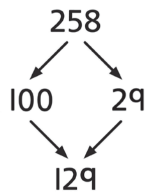


These methods can be used to solve word problems.

Halving

Children find half of even numbers to 100 using partitioning. They move beyond 100 in year 4.



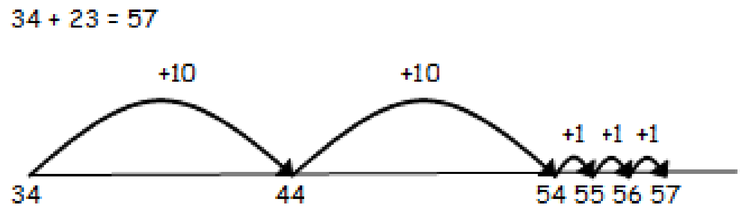


Empty number lines

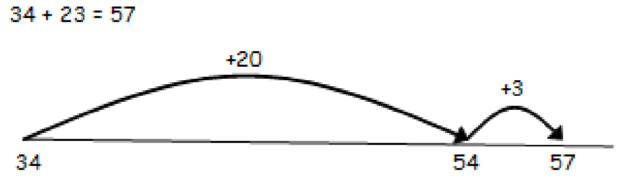
Year 2

Children use empty number lines, equipment, hundred squares, Diennes, Numicon to build fluency in metal addition skills.

Children start with the largest number and count on in tens and then ones.



The next step using this method is to count the tens in one jump and the units in one jump.



As children build up confidence with using a number line the numbers get larger and bridge the hundreds.

Children often spend a lot of time consolidating this stage and need lots of practise counting past 100 in tens.

Expanded column addition

Year 2/3

To use this method children need to be confident with partitioning.
Firstly they partition the numbers in the calculation and lay out in columns.
At first numbers which do not cross the tens and units boundaries are used.

This method can be supported using Diennes and Numicon.

34 + 43 = 77

$$\begin{array}{r} 30 + 4 \\ +\quad 40 + 3 \\ \hline 70 + 7 \\ \hline \end{array}$$

39 + 43 = 82

$$\begin{array}{r} 30 + \ 9 \\ +\quad 40 + \ 3 \\ \hline 70 + 12 \\ \hline \end{array}$$

39 + 43 = 82

$$\begin{array}{r} 30 + 9 \\ +\quad 40 + 3 \\ \hline 80 + 2 \\ \hline 10 \end{array}$$

$$\begin{array}{rrrr} & 400 & 60 & 6 \\ + & 300 & 50 & 8 \\ \hline & 700 & 110 & 14 \\ \hline \end{array} = 824$$

$$\begin{array}{rrrr} & 400 & 60 & 6 \\ + & 300 & 50 & 8 \\ \hline & 800 & 20 & 4 \\ \hline & 100 & 10 & \end{array}$$

Grouping and sharing

Year 1/2

Grouping

Relate division to multiplication by using arrays or towers of cubes to find answers to division

e.g. *'How many towers of five cubes can I make from twenty cubes?' as _ × 5 = 20 and also as 20 ÷ 5 = _*

Relate division to 'clever' counting and hence to multiplication

e.g. *'How many threes do I count to get to nine?'*

Children can use Numicon to help with grouping.

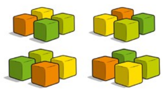
Sharing

Begin to find half of a quantity using sharing

e.g. *find half of 12 cubes by giving one each repeatedly to two children*

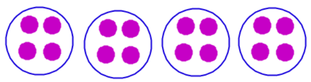
Begin to find half or a quarter of a quantity using sharing

e.g. *find a quarter of 16 cube by sorting the cubes into four piles*



In year 2 children continue to share objects into groups but now should recognise a division calculation.

16÷4 = 4



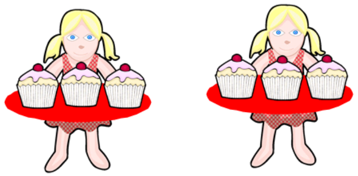
Grouping and sharing in play

Year R

In reception children begin to share through everyday situations.
For example:

- Sharing food at snack time
- In role play situations
- Sharing equipment in the class

They will use a range of practical objects, for example food, toys and pencils.



Children also find half of a number of objects by sharing them into two equal groups using practical resources.

Compact column addition

Year 3/4/5/6

Once secure in the previous method children will begin to carry digits below the line.
It is important that children have a secure understanding of place value.

Children will then extend this method throughout years 4, 5 and 6 by:

- Adding several numbers with different numbers of digits.
- Using column addition to add up money making sure the decimal point aligns.
- Using column addition to solve measure calculations including those with decimals.

Adding together H T U

$$\begin{array}{ccccc} & & 5 & 8 & 9 \\ + & & 4 & 5 & 3 \\ \hline & 1 & 0 & 4 & 2 \\ \hline & & \cancel{1} & \cancel{1} & \end{array}$$

Adding together Th H T U

$$\begin{array}{ccccc} & 5 & 1 & 8 & 9 \\ + & 2 & 5 & 3 & 1 \\ \hline & 7 & 7 & 2 & 0 \\ \hline & & \cancel{1} & \cancel{1} & \end{array}$$

Adding several numbers and using 0 as a placeholder

$$\begin{array}{cccc} & 15. & 8 & 0 \\ + & 2. & 5 & 7 \\ + & 1. & 1 & 0 \\ \hline & 19. & 4 & 7 \\ \hline & \cancel{1} & & \end{array}$$

It is very important that when recording this method, the columns line up.

Once your child is secure in understanding how this method works, they should be able to apply their knowledge in a range of situations and to use this strategy to support problem solving involving decimals e.g. money.

$$\begin{array}{rr} & 4.78 \\ & 2.03 \\ + & 0.63 \\ \hline & 7.44 \\ \hline & {\scriptstyle 1\ 1\ \ } \end{array} \qquad \qquad \begin{array}{rr} & £4.78 \\ & £2.03 \\ + & £0.63 \\ \hline & £7.44 \\ \hline & {\scriptstyle 1\ 1\ \ } \end{array}$$

This may involve changing pence to pounds so that all the values are represented in the same way before the addition is calculated e.g. 63p becomes £0.63 – otherwise there could be confusion!

$$\begin{array}{rr} & £\ 5.85 \\ & £\ 0.99 \\ + & £\ 11.45 \\ \hline & £\ 18.29 \\ \hline & {\scriptstyle 2\ 1\ \ } \end{array}$$

Dynamic Division

Short multiplication HTU x U and decimals

Year 4/5/6

Once the key steps for the grid method have been mastered, calculation is done using the more familiar method starting with the least significant digit first:

$$\begin{array}{r} 126 \\ \times \quad 6 \\ \hline 756 \\ \hline {\scriptstyle 1\ 3\ } \end{array}$$

Long multiplication HTU x TU

Multiplication by two digit numbers comes next. Again, start by multiplying the least significant digit first. Place value is very important throughout, especially when multiplying by the tens digit.

$$\begin{array}{r} 1\ 3\ 4 \\ \times \quad 2\ 7 \\ \hline 9\ 3\ 8 \\ 2\ 6\ 8\ 0 \\ \hline 3\ 6\ 1\ 8 \\ \hline \end{array}$$

Children continue to use long and short multiplication to multiply decimals with up to 2 decimal places by a single digit.

$$\begin{array}{r} \pounds\ 1\ 3\ .\ 7\ 2 \\ \times \qquad\qquad 6 \\ \hline \pounds\ 8\ 2\ .\ 3\ 2 \\ \hline {\scriptstyle 2\ \ 4\ \ 1\ \ } \end{array}$$

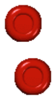
Super Subtraction

Starting out

Children count backwards in ones.

They begin to identify one more and one less than a given number.

Children use various objects to visually support subtraction calculations by physically taking away objects.

 - 1 = 

They can also use their fingers to count back on after they have 'put the number in their head'

-  = 4

Grid method (TU xTU)

When children have fully grasped multiplication by one digit, this is further extended to multiplying by two digits in the following way: **Both** numbers are partitioned.

25 x 16 =

×	10	6		
40	400	240	=	640
8	80	48	=	128
				768

These methods are extended to be used with decimals. This needs to be done with care so that place value is firmly kept in mind throughout. It might mean converting pence to pounds or vice versa - calculations should be recorded consistently:

£1.34 x 6 =

×	1	0·3	0·04		
6	6	1·8	0·24	=	8·04

Ladder Method

Year 4

Once grid multiplication has been mastered, children are taught the 'ladder method' which is an expanded form of short multiplication of TU x U and HTU x U.
Children will be taught this method alongside the grid method initially so children can see the connection.

Children will be taught to solve HTU x U using the ladder method too.

```
      2 5 3
×         6
  ---------
    1 2 0 0  ←— 6 × 200
      3 0 0  ←— 6 × 50
+       1 8  ←— 6 × 3
  ---------
    1 5 1 8
  ---------
```

This is the expanded form of short multiplication which the children will master later. If children are confident short multiplication will be taught.

Grid multiplication will be used until children can master short multiplication

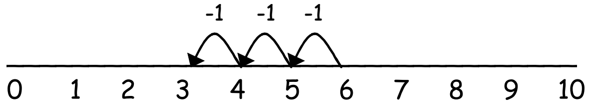
Counting back on number lines

Children begin to understand subtraction as **'take away'** and recognise that when doing this you end up with **less**.
They continue to consolidate their understanding of subtraction practically using bead strings, Numicon, cubes and are gradually introduced to the number line.

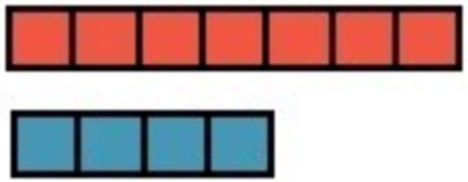
Children move onto using number lines to count back in ones initially under ten, then bridging ten, then higher numbers.

Children learn about the 'difference'. Children often find it hard

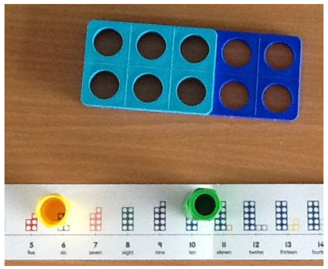
6 - 3 = 3



to relate this phrase to subtraction.
Children can also use cubes and Numicon to show the difference between by laying cubes next to each other or Numicon on top of each other.



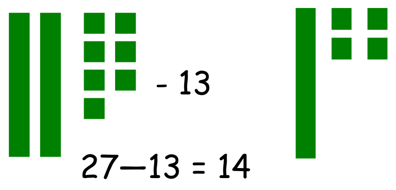
Difference between 7 and 4 is 3.



Overlay the Numicon to show the difference between 10 and 6 is 4.

Using Diennes and Numicon

Year 1/2

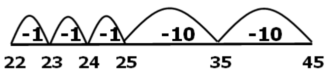


Numbers which bridge 10 when taking away e.g 43—19 can still be calculated using Diennes but a ten needs to be exchanged for ten ones.
Numicon can be used by overlaying the pieces as explained before to find the difference.

Counting back on number lines

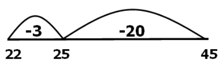
Partition the second number and subtract it in tens and units.

45 - 23 =



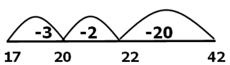
Move towards more efficient jumps back.

45 - 23 = 22



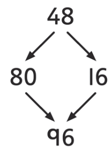
Bridge through ten to help them become more efficient.

42 - 25 = 17



Doubling numbers

Year 3/4



Children use partition to help double numbers

Grid Method (TU x U)

x	20	3
4	80	12

= 92

Once basic multiplication by one digit has been mastered, this is extended to two digits. This is done using a grid in which the two digit number has been **partitioned**.
Children start by multiplying numbers by 3, 4 and 5 and extend to larger numbers. Children can always jot a number line at the side to calculate the u x u calculation.

The GRID method (HTU x U)

The next stage is to extend multiplication of 3-digit numbers by 1-digit in the same way. Always multiply by the smaller number.
6 x 253 or 253 x 6 will be recorded as:

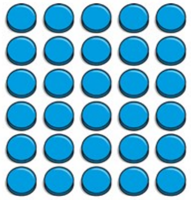
x	200	50	3
6	1200	300	18

= 1518

Arrays

Year 1/2

Children use jottings to help them find the answer. These are called arrays.



5 x 6 = 30

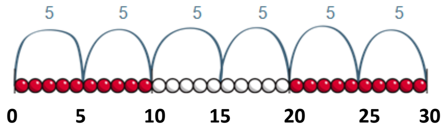
5 x 6 = 5+5+5+5+5+5=30
6 x 5 = 6+6+6+6+6 = 30

6 x 5 = 30

This allows children to understand that multiplication can be done in either order.

Repeated addition on a number line

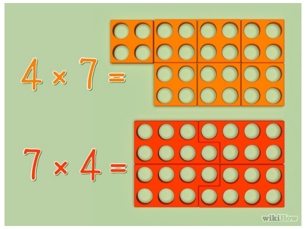
Repeated addition can be shown easily on a number line.
6 x 5 = 30



Children must learn to apply this knowledge in practical problem solving activities. Children will continue using this method into year 3 if necessary.

Numicon

Numicon can be used to multiply practically.



Counting on using a number line (frog maths)

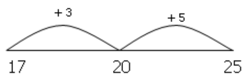
Year 2/3/4

Children begin to use more mental methods to calculate subtraction. One of these is to count on from the smallest number to the largest number to find the difference.

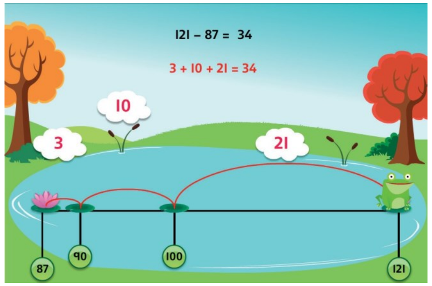
It is much easier for children to count on than count back.

Children are then taught how to use this for larger calculations on a blank number line. Children might also need to use a 100 square or other labelled number line to help them visualise counting on.

25 − 17 = 8

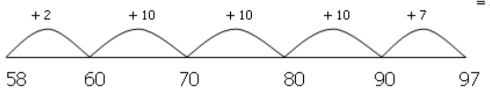


3 + 5 = 8



At first children use counting on to solve small differences. They count up from the smaller number to the larger number and add up the jumps to calculate the difference.

97 − 58 = 39

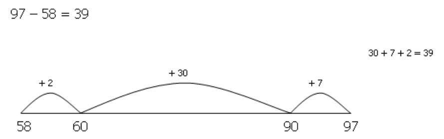


10 + 10 + 10 + 7 + 2
= 39

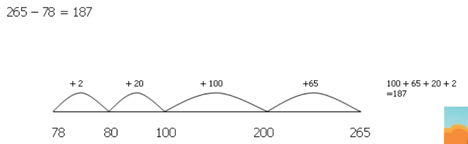
Children are taught to count to the nearest 10 first.
As they become more confident they are able to tackle larger numbers. First counting on in lots of 10s then...

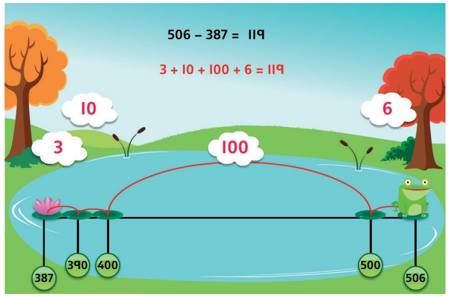
... counting on the tens as a big jump.

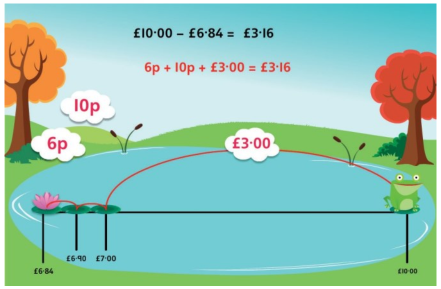
Year 3/4



Children use the same method for larger numbers and money.







Children will continue to use counting up to solve mental calculations, change questions from whole amounts (e.g £10.00—£8.78) and decimals (1.2-0.87), as they progress through upper KS2

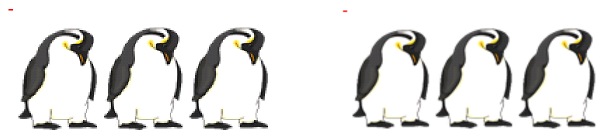
Doubling, groups and counting in steps

Year 1

Children continue counting in different steps both forwards and backwards e.g. 2s and 10s up to 100 and begin counting in 5s.

They learn to mentally recall doubles of numbers up to, at least, double 5. Children need to be able to recall these facts quickly.

They solve problems e.g There are 3 penguins at the zoo. If the number of penguins doubled, how many penguins would there be?



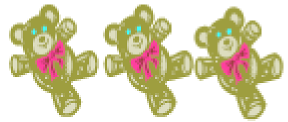
Numicon counting in steps

4 + 4 = 8

Using pictures and practical objects

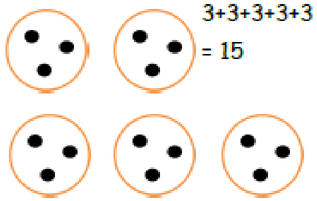
Children will develop their understanding of multiplication and use jottings to support their calculations.
Children begin to understand multiplication as repeated addition e.g. 3 x 2   3 lots of 2   2 + 2 + 2

How many legs will 3 teddies have?



2 + 2 + 2 = 6

There are 3 sweets in one bag.
How many sweets are in 5 bags altogether?



A note about times tables

Learning rapid recall of their times tables is an important development in children's mathematical understanding. Children can start to learn their times tables when they are ready for the concept.

However, as a guideline, the tables children **should** be able to recall quickly and accurately up to x 12 by the end of each year:
Year 1: Count in 2s, 5s and 10s
Year 2: 2x 5x 10x
Year 3: 3x 4x 8x
Year 4: All multiplication tables up to 12 x 12
Children should also be able to recall the related division facts for the times table they are learning.

Groups of objects

Year R

Children in reception learn to count repeated groups of the same size. E.g. counting pairs of socks

They will also learn simple doubles using groups of objects.

Expanded column subtraction

Year 4/5

The first stage of decomposition is to partition the number into H T U first then take the smaller number from the larger number. The children will record the calculation like this and start with numbers which do not require exchanging.

89 - 35 = 54

	80	9
-	30	5
	50	4

Then introduce exchanging through practical subtraction. Use Diennes to make a number then subtract a number from it, exchanging as necessary.
For example before subtracting 47 from 72 children need to exchange a ten for ten units. They can then subtract 7 and 40.

72 - 47 = 25

	60	12
	~~70~~	~~2~~
-	40	7
	20	5

Once pupils are secure with the understanding of 'exchanging' they can use the partitioned column method to subtract bigger numbers. They will gradually progress to compact column subtraction when confident.

Expanded

	600	110	16
	~~700~~	~~20~~	~~6~~
–	300	50	8
	300	60	8

Compact

	6	11	16
	~~7~~	~~2~~	~~6~~
–	3	5	8
	3	6	8

Compact column subtraction

Year 5/6

When children have fully grasped why we 'exchange' from tens to units or hundreds to tens, they are ready to omit some of the recorded steps to make it quicker.

They are introduced to the compact method by completing a partitioned column subtraction then being shown the compact method beside it.

	6	14	1
	~~7~~	~~5~~	4
-		8	6
	6	6	8

Step 1:

		4	1
	7	~~5~~	4
-		8	6
			8

(adjust from T to U)

Step 2:

	6	14	1
	7	~~5~~	4
-		8	6
		6	8

(adjust from H to T)

Step 3 :

	6	14	1
	7	~~5~~	4
-		8	6
	6	6	8

As children move through the school they use decomposition with increasingly large, more complex and decimal numbers.

	0	15	13	1	14
	~~1~~	~~6~~	~~3~~	~~2~~	~~4~~
-		8	5	1	6
		7	8	0	8

	7 1
	£ 56~~8~~.49
-	£ 127.53
	£ 440.96

	2	14		7	15
	~~3~~	~~4~~	6	~~8~~	~~5~~
-	1	6	4	5	8
	1	8	2	2	7

Marvellous Multiplication